Math Problem-Solving Story Mats

The Mentoring Minds *Math Problem–Solving Story Mats* are designed for interactive problem solving use. The number and math symbol cards facilitate the translation of words to numbers and symbols. This critical connecting step helps students bridge the gap between the concrete and the abstract. This product is designed for use with partners, in small groups, in centers, or with whole group instruction. Math problem-solving story mat materials can be purchased as a *Math Problem-Solving Story Mats Blackline Master*, as *Math Problem-Solving Story Mats Transparencies*, or as a *Deluxe Problem-Solving Story Mats Kit*.

A review of literature indicates concerns from politicians to researchers, educators, and people in general. They express that too few students in elementary and middle schools are successfully acquiring the mathematical knowledge, the skill, and the confidence they need to use the mathematics they have learned. The mathematics students need to learn today differs from mathematics that students needed to learn in the past. When today's students become adults, undoubtedly they will encounter new demands for mathematical proficiency (Kilpatrick, Swafford, Findell, 2001). What does remain the same is the effect of teaching for meaning and understanding. Studies have consistently shown that when an emphasis is placed on teaching for meaning, positive effects on student learning result. These studies can be traced to the early findings of William Brownell (1945, 1947).

Today's students are growing up in a world surrounded by mathematics. Households, schools, and workplaces all utilize technology built on mathematical knowledge. Many educational opportunities and quality jobs require high levels of mathematical expertise. Mathematical topics occur in the media, in literature, in entertainment, and in everyday conversation.

Mathematics is a universal subject, so much a part of life that anyone who is a participating member of society must know basic mathematics. Students' mathematical achievement, however, is ultimately determined and limited by the opportunities they have had to learn. Mathematics is not restricted to a select group of students. "All students must learn to think mathematically, and they must think mathematically to learn" (Kilpatrick, Swafford, Findell, 2001).

Hiebert and Wearne (1992, 1993, 1996) reported that a critical attribute in regards to student learning in mathematics, is the nature of the learning task in which to engage students. Students need mental engagement in challenging and worthwhile mathematical tasks that emphasize the conceptual aspects of the topic and promote the formation of mathematical connections. This type engagement is a prerequisite to learning skills with meaning and being able to apply those skills to solve problems. Students must receive direct encouragement to think and persist with the mathematical task at hand.

Grouws and Cebulla (2000) state that teaching mathematics with a focus on number sense encourages students to become problem solvers in a wide variety of situations and to view mathematics as a discipline in which thinking is important. "Number sense" is an intuitive feel for number size and combinations, as well as the ability to work flexibly with numbers in problem situations in order to make reasonable judgments. The processes of mentally computing, estimating, sensing number magnitudes, moving between representation systems for numbers, and judging the

reasonableness of numerical results must be flexibly used. This type of instruction requires teachers to have a deep understanding of mathematics and how students learn mathematics. More specifically, teachers will encounter difficulty in teaching number sense without a working knowledge of number sense themselves.

Students tend to view word problems as a task that causes anxiety in the mathematical classroom (Kouba, Brown, Carpenter, Lindquist, Silver, Swafford, 1988). Word problems appear to be difficult for many students which indicates a need to address the situation because these problems play a prominent role in mathematics instruction. When products are developed to address this and reflect other research findings, the impact on student performance can be favorable.

An open-ended problem is presented for students to solve using words, numbers, or pictures and to follow up with a written explanation. A basic characteristic needed to become a proficient problem solver is flexibility. Flexibility develops through the expansion of knowledge required for solving nonroutine problems rather than just routine problems.

Nonroutine problems (those not familiar to the problem solver) and transfer of problem solving require high level transfer, which is effortful and conscious (Salomon & Perkins, 1989), whereas routine problems involve less conscious attention and rely more on low level transfer. Routine problems are those in which the learner knows a correct solution method based on past experience and is able to reproduce it and apply it. Caution, given by experts, is that students can lose the ability to articulate and reflect on the reasoning they use in solving problems if they are exposed to mostly routine problems. Nonroutine problems require the learner to use productive thinking to create a way to understand and solve the problem since an immediate solution method is not known. A balance is needed between the time students spend practicing routine procedures and the time they devote to discovering new method solutions for nonroutine problems. There is no need for the teacher to make a choice between these two type problems. The *Problem Solving Story Mats Kit* incorporates routine and nonroutine problems in the Instructional Resource Book because both are essential for students to develop mathematical thinking power.

Like most skills in mathematics, problem solving requires practice. Often students read word problems in mathematics with no comprehension of the action in the problem. To address this issue, Mentoring Minds developed the story mat kit to help students connect words with actions and translate those actions into the symbols of mathematics. The problem-solving strategy, acting it out with models, is practiced extensively with the kit. This strategy provides a concrete foundation for solving word problems so that students can move easily into other problem-solving strategies and models with conceptual understanding.

Kulm (1984) indicated that problem context could assist students in gathering meaning in a word problem. Bickmore-Brand (1993) found that context provides a basis to connect meaning to any mathematical activity. The foundation for the story mats lies in real-world problems with a connection to science. Due to students' natural curiosity of science, each of the six story mats relates to a different habitat. The bank of problems for each story mat design ranges from one-step problems to two-step or multi-step problems with mixed operations and with and without extra information.

The story mats provide a concrete foundation for students to have hands-on practice in solving word problems. The use of manipulatives and pictorial materials are considered paramount in teaching mathematics. Grouws and Cebulla (2000) state that long-term use of concrete materials is positively related to increases in mathematics achievement and improved attitudes toward mathematics. In the report, "Improving Student Achievement in Mathematics," findings suggest teachers use manipulative materials in mathematics instruction regularly in order to provide students hands-on experience that enables them to construct useful meanings for the mathematical ideas they are learning. Furthermore, connections between ideas are formed and strengthened. The use of concrete materials should not be limited to demonstrations. It is essential that students use materials that stimulate thinking in meaningful ways. Students must see the relationship between the concrete embodiments of a math concept and the notational system used to represent it. The story mats allow the students to solve problems with manipulatives moving from the concrete to connecting with the pictorial and the abstract. The teacher can use the matching story mat transparencies to demonstrate, introduce, clarify, or reinforce the mathematical concept being addressed.

In addition, the use of the story mats facilitates classroom discussion to promote student understanding. Using a variety of group activities is essential in mathematics instruction. Grouws and Cebulla (2000) found that using small groups of students to work on activities, problems, and assignments can increase student mathematics achievement. These researchers noted that using whole-class discussion following individual and group work improves student achievement.

The use of the story mats, designed for a partner format, promotes ongoing student discussion and facilitates classroom discussion following the partner work time. This process increases students' mathematical understanding. Initially, students read and hear the word problem and then use counters to act out the story problem on the mat. While simultaneously creating pictorial representations of the story problem, the students build a symbolic representation of what they acted out at the bottom of the story mat. This representation connects the action in the word problem with the mathematical symbols.

Students talk about how they solved the problem with their partners. When the entire class reconvenes, the teacher follows up the problem-solving activity with a class discussion so that all students see there can be different ways to solve problems. It is essential that the reasoning of other students is shared. Although the reasoning might be different, an equivalent symbolic representation is reached and students need to hear and see this important step in problem solving.

Research reflects the importance of whole-class discussion following student work on problem-solving activities. Findings indicate that such discussion following individual and small group work improves student achievement (Grouws and Cebulla, 2000). The discussion includes a summary of key points of individual work. This can be accomplished through students presenting and discussing their individual solution methods, or through other methods of achieving closure that are led by the teacher, the students, or both. When students have opportunities to listen to and share their thinking with their peers, they become more reflective about their work and increase their mathematical understandings. As a result, students learn to apply and adapt a variety of appropriate strategies to solve problems.

Teachers can also employ whole-class discussion as a student diagnosis tool for identifying areas of difficulty, for determining misconceptions, and for ascertaining areas of student success or progress. According to the National Council of Teachers of Mathematics (2000), assessment is a crucial component in mathematical achievement. Evidence for assessing problem solving is collected by observing students as they work and listening to students as they discuss and explain their thought processes for arriving at a solution. This information gathered can guide teachers as they effectively plan for meaningful mathematics instruction.

Utilization of the *Math Problem-Solving Story Mats* enhances student use of the language of mathematics. Connections are formed for and by students between words in the word problems and the actions and symbols that represent the word problems. As students use manipulatives and counters to build a model of the problem on the story mat, they use mathematical language to talk about this as they work. In conclusion, students build equations or number sentences with symbol cards to record the action in the problem and share this orally.

Teachers must ensure that ample opportunities for students to learn important content skills are provided. If students are to compete in a technologically-focused society, they must be taught the mathematical skills to do so. Therefore, if problem solving is crucial, which it is, then a specific focus must be given to it on a regular basis. Evidence from research demonstrates that a successful mathematics program must include time for students to practice what they are learning and experiences to perform the tasks for which they are to demonstrate competence. Often, students appear to spend more time on skill work rather than developing problem solving and higher-order thinking abilities (Boaler, 1998; Stigler and Hiebert, 1997; Wood and Sellers, 1996, 1997).

The Mentoring Minds' Product Development Team sought to develop a mathematics resource to assist teachers as they present students with active learning experiences to better relate to problem solving. The *Problem-Solving Story Mats Blackline Master* is written so teachers could easily replicate the format and personalize the context of the problems with individual student names or other pertinent yet personal information. Studies show the interest and motivation level of students appear to show favorable results when students solve personalized problems as evidenced in several studies (Davis-Dorsey, Ross, and Morrison, 1991; Hart, 1996; Lopez and Sullivan, 1991, 1992). Problems which connect to classroom happenings or to events from the lives of the students bring relevance to the mathematical situations for students. By solving such problems, students apply mathematical concepts in authentic contexts to develop new mathematical understandings.

The resulting product, the *Math Problem-Solving Story Mats*, provides teachers a bank of word problems that correspond to each story mat, ranging from simple, one-step problems to complex, multi-step problems. Transparencies are included so that teachers may model the procedure for problem solving, present the purpose of each problem, follow-up by demonstrating the concept or clarifying any confusion, and to reinforce the skills students need for mathematical proficiency.

Bibliography for Math Problem-Solving Story Mats

Bickmore-Brand, (1993). I*mplications from recent research in language arts for mathematical teaching.* In J. Bickmore-Brand (Ed.), Language in Mathematics.

Portsmouth, NH: Heinemann.

Boaler, J. (1998). Open and closed mathematics: student experiences and understanding. *Journal for Research in Mathematics Education*, 29, 41-62.

Brownell, W. (1945). When is arithmetic meaningful? *Journal of Educational Research*, 38, 481-498.

Brownell W. (1947). The place of meaning in the teaching of arithmetic. *Elementary School Journal*, 47, 256-265.

Davis-Dorsey, J., Ross, S. , & Morrison, G. (1991). The role of rewording and context personalization in the solving of mathematical word problems. *Journal of Educational Psychology*, 83, 61-68.

Grouws, D. & Cebulla, K. (2000). *Improving Student Achievement in Mathematics*. Geneva, Switzerland: International Academy of Education International Bureau of Education, Educational Practices Series -4.

Hart, J. (1996). The effect of personalized word problems. *Teaching Children Mathematics*, 2, 504-505.

Hiebert, J. & Wearne, D. (1992). Links between teaching and learning place value with understanding in first grade. *Journal for Research in Mathematics Education*, 22, 98-122.

Hiebert, J. & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal*, 30, 393-425.

Hiebert, J. & Wearne, D. (1996). Instruction, understanding, and skill in multi-digit addition and subtraction. *Cognition and Instruction*, 14, 251-283.

Kilpatrick, J., Swafford, J., Findell, B. (Eds.), Mathematics Learning Study Committee, National Research Council (2001). Conclusion and recommendation. In *Adding It Up: Helping Children Learn Mathematics*. Washington, D.C. : The National Academies Press.

Kouba, V., Brown, C., Carpenter, T., Lindquist, M. Silver, E. , & Swafford, J. (1988). Results of the Fourth NAEP assessment of mathematics: number, operations, and word problems. *Arithmetic Teacher*, 34, 14-19.

Kulm, G. (1984). The classification of problem-solving research variables. In G.A. Goldin & C.E. McClintock (Eds.), *Task Variables in Mathematical Problem Solving.* Philadelphia: The Franklin Institute Press.

Lopez, D. & Sullivan, H. (1991). Effects of personalized math instruction for Hispanic students. *Contemporary Educational Psychology*, 16, 95-100.

Lopez, D. & Sullivan, H. (1992). Effects of personalization of instructional context on the achievement and attitudes of Hispanic students. *Educational Technology Research and Development*, 40, 5-13.

Principles and Standards for School Mathematics. (2000). Reston: VA, National Council of Teachers of Mathematics.

Salomon, G., & Perkins, D. (1989). Rocky roads to transfer: Rethinking mechanisms of a neglected phenomenon. *Educational Psychologist, 24*, 113-142.

Stigler, J., & Hiebert, J. (1997). Understanding and improving classroom mathematics instruction. *Phi Delta Kappan*, 79, 14-21.

Wood, T., & Sellers, P. (1996). Assessment of a problem-centered mathematics program: Third grade. *Journal for Research in Mathematics Education*, 27, 337-353.

Wood, T., & Sellers, P. (1997). Deepening the analysis: longitudinal assessment of a problem-centered mathematics program. *Journal for Research in Mathematics Education*, 28, 163-186.