

Mentoring Minds' Research on Math Essentials

Math Essentials is a comprehensive supplemental resource for the development of essential math skills through an open-ended response format. Constructed-response practice pages are included for Number and Operations, Algebra, Geometry, Measurement, Statistics and Probability, and Problem Solving. These strands are identified as the scope of mathematical content areas found within the National Assessment of Educational Progress Mathematics Assessment framework (NAEP, 2005).

Mathematics is a universal subject, so much a part of life that anyone who is a participating member of society must know basic mathematics. Students' mathematical achievement, however, is ultimately determined and limited by the opportunities they have had to learn. Mathematics is not restricted to a select group of students. "All students must learn to think mathematically, and they must think mathematically to learn" (Kilpatrick, Swafford, and Findell, 2001). The RAND Mathematics Study Panel (2003) also emphasized the importance of mathematics. The panel declared that it is essential that students develop math proficiency. The No Child Left Behind Act (NCLB, 2001) directed that research about effective practices be a guide in changing the way mathematics is taught so that high standards are met leading to an improvement in achievement.

The National Council of Teachers of Mathematics (NCTM) is a leading professional organization for K-12 teachers of mathematics. This organization is responsible for providing guidance in areas related to mathematics. A set of standards for mathematics, *Principles and Standards for School Mathematics*, were developed to serve as guidelines for teachers, school, districts, and states (NCTM, 2000). Teachers of mathematics, whether at elementary or secondary levels, should be aware of the national, state, and local mathematics standards and principles in addition to being cognizant of best practices in teaching and assessment.

For students to become mathematically literate, they must have opportunities to:

- Explore mathematics as indicated in the NCTM *Principles and Standards for School Mathematics*
- Develop a thorough understanding of mathematical content
- Link their understanding of concepts to multiple forms of representation (such as, diagrams, graphs, tables, manipulatives, and symbolic expressions) (Palacios, 2005).

The National Assessment for Educational Progress (NAEP), also known as "the Nation's Report Card," reports results for subject-matter achievement. NAEP also surveys teachers and students relating to their school experiences. The National Center for Educational Statistics (NCES) uses the NAEP to monitor student

achievement of American students through random-sampling testing of Title I schools using the NAEP test. NCLB legislation mandates that any Title I school selected must agree to administer the test. Implementation changes in mathematical assessments for grades 4, 8, and 12 were made by NAEP (1992) based on recommendations from the Standards and other mathematical achievement data across the nation. NAEP Mathematics tests now include questions and tasks that ask students to construct individual responses in two ways. A short-constructed question is posed that requires a short response. An extended-construction response task is given that requires students to solve problems at a higher level of thinking. Students then share their mathematical reasoning and problem-solving abilities through a written explanation of the solution's key points (ETS, 1994).

In 1992, NAEP mathematics assessment was at a lower level of performance on both regular and extended-constructed response questions than on the multiple-choice items. The average students' performance reflected an increase on multiple-choice formatted questions. The Office of Education Research and Improvement (OERI), summarized the gathered NAEP data as follows. Approximately one third to two-thirds of the students provided incorrect responses to extended-response questions indicating little evidence of understanding the mathematical concepts involved or the questions being asked. Most students who appeared to understand the problems had difficulty explaining their work (ETS, 1994). The need for effective instruction and practice involving open-ended mathematical problems was evident.

NAEP's results showed significant gains in basic math skills but not in higher order thinking skills. The discouraging results were partially attributed to the influence of standardized testing on teaching and learning according to Shepard and Dougherty (1991). Stodolsky (1988) indicated concern had surfaced about mathematics instruction in the nation's schools due to the level of student achievement in multiple choice items as opposed to extended response items. Another finding indicated that mathematical problems emphasize number operations through individual seatwork and recitation with little attention given to group work, discussion or problem solving.

Classroom observations from a study of three schools (NAEP, 1992; 1994) identified the textbook as having the largest influence on mathematics instruction. Lindquist (1997) indicated that instruction depended on the text and the lecture method. Generally, when the mathematics curriculum in the schools was based on skill and drill, educational experiences failed to support those guidelines outlined in the NCTM Standards. Furthermore, the findings showed students were seldom asked to explain their thinking, justify their answer, or give a detailed response. Students rarely formulated a problem for themselves. Instructional formats appeared to center around teacher talk and independent work. Usually, the learning focused on mastery of isolated skills with little connectedness between mathematical ideas and concepts. Time spent in

activities that engaged higher levels of thinking, mathematical reasoning, or problem solving was minimal. The teaching and learning of mathematics centered on recall, basic facts, and skill acquisition.

The 1996, 2000, and 2003 national mathematics assessments focused on reasoning and communication. Students were asked to connect their learning across the mathematical strands. The 2003 NAEP results have been encouraging as they show improvement in overall mathematics scores, particularly at fourth grade (NCES, 2005). This is an important finding due to rising emphasis of the standards movement. Previously elementary students received much exposure to computation whereas teachers now have access to all NCTM standards. Stevens (2003) interpreted the results as progress, yet still voiced concern about moving from the Basic level to the next level of Proficiency. His concern centered on the ability of students to apply the mathematical skills to unfamiliar situations, to formulate their own problems and to solve those given.

Hiebert and Wearne (1992; 1993; 1996) reported that a critical attribute in regards to student learning in mathematics is the nature of the learning task in which to engage students. Students need mental engagement in challenging and worthwhile mathematical tasks that emphasize the conceptual aspects of the topic and promote the formation of mathematical connections. This type of engagement is a prerequisite to learning skills with meaning and being able to apply those skills to solve problems. Students must receive direct encouragement to think and persist with the mathematical task at hand.

As targeted in the standards, the role of the mathematics teacher is to solicit student thinking by posing problems, asking questions, and encouraging exploration of solutions (NCTM, 2000). Another recommendation is for students to share their learning processes with fellow students (NCTM, 2000). The Mentoring Minds' Product Development Team encourages teachers to incorporate these two recommended practices from NCTM as students work through the practice pages of *Math Essentials*. Transparency Notebooks of all pages are also available. Transparencies are included so that teachers may present the purpose of each problem, model the procedure for problem solving, provide follow-up by demonstrating the concept or clarifying any confusion, stimulate student discussion on solution methods, or offer reinforcement of the skills students need for mathematical proficiency.

Teachers must ensure that ample opportunities for students to learn important content skills are provided. If students are to compete in a technologically-focused society, they must be taught the mathematical skills to do so. Therefore, if problem solving is crucial, which it is, then a specific focus must be given to it on a regular basis. Evidence from research demonstrates that a successful mathematics program must include time for students to practice what they are learning and experiences to perform the tasks for which they are to demonstrate competence. A balance should be given between computation and using

mathematics to solve problems. Often, students appear to spend more time on skill work rather than developing problem solving and higher-order thinking abilities (Boaler, 1998; Stigler and Hiebert, 1997; Wood and Sellers, 1996, 1997). Using open-ended problems, *Math Essentials* offers an opportunity to provide the essential element of balance.

Whole-class discussion can be utilized by teachers as a student diagnostic tool for identifying areas of difficulty, for determining misconceptions, and for ascertaining areas of student success or progress. According to the National Council of Teachers of Mathematics (2000), assessment is a crucial component in mathematical achievement. Evidence for assessing open-ended problems is collected by observing students as they work through *Math Essentials* and listening to students as they discuss and explain their thought processes for arriving at solutions. The information gathered can guide teachers as they effectively plan for meaningful mathematics instruction.

Research reflects the importance of whole-class discussion following student work on problem-solving activities. Findings indicate that such discussion following individual and small group work improves student achievement (Grouws and Cebulla, 2000). The discussion includes a summary of key points of individual work. This can be accomplished through students presenting and discussing their individual solution methods or through other methods of achieving closure that are led by the teacher, the students, or both. When students have opportunities to listen to and share their thinking with their peers, they become more reflective about their work and increase their mathematical understandings. As a result, students learn to apply and adapt a variety of appropriate strategies to solve problems in *Math Essentials*.

“Assessment plays a critical role in all aspects of teaching and learning mathematics” (NCTM, 2000). In the publication compiled from numerous writers, No Child Left Behind Act (NCLB, 2001) stated, “Beginning no later than the 2005-2006 school year, each State must administer annual assessments in math in each of grades 3 through 8 ...” The state assessment scores are also used to measure adequate yearly progress (AYP) for all public schools. All students’ scores are part of the AYP measure, including students with disabilities and limited English proficiency. The accountability measure and the need for higher standards in mathematics make it crucial to ensure students become mathematically literate. The practice pages in *Math Essentials* provide teachers with information to detect students’ misunderstandings in mathematical concepts and to determine the extent of students’ grasp of mathematical knowledge.

The purpose of the Mentoring Minds’ Product Development Team was to develop a product that meets the national mathematical standards and reflects research findings. The literature revealed that while the number of American students who could demonstrate mastery of basic mathematics facts was on the rise, many are not proficient in using this knowledge to solve everyday problems. A resulting

product is *Math Essentials* which contains open-ended problems. Short constructed-response questions require students to provide answers to computation problems or to describe solutions in one or two sentences. Extended constructed-response questions require students to give longer responses using words and/or diagrams to demonstrate their conceptual and procedural knowledge when answering the questions. *Math Essentials* is available in three formats: Student Edition, Blackline Master, and Transparency Notebook.

Math Essentials can assist teachers as they develop mathematical literacy by integrating open-ended formats into classroom practice for students. Questions of this nature promote higher level thinking opportunities for students. Thus, *Math Essentials* offers support to help improve mathematics instruction and to help meet the accountability outlined by the NCLB (2001) legislation and by NCTM (2000).

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