Product Research

ThinkUp!™ MATH

Supporting documentation for the development of **ThinkUp! Math**

Mentoring Minds®

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ThinkUp! Math

To fully prepare students in Texas for success in college, in the workplace, and life in the 21stcentury, rigorous academic standards were developed. These standards place a focus on improving student achievement and go beyond fundamental knowledge and skills. The Texas Essential Knowledge and Skills (TEKS) promote increased accountability in education. Educators may examine the Standards for Mathematics (TEA, 2014a) to improve the *what* and *how* of instruction. While these standards identify what students are to know and be able to do, the how decision remains with individual districts, schools, and teachers. Schools in Texas must align standards, instruction, and assessments with 21st century skills. Furthermore, students must learn how to apply these skills in the context of the real world. Research indicates that students understand and retain more when learning is relevant, engaging, and meaningful to their lives. Instruction that focuses on the TEKS demonstrates rigor and relevance in today's classrooms. *ThinkUp! Math* was developed specifically around the Mathematics Standards for Texas and serves as a resource to support teachers in the implementation of mathematics education.

ThinkUp! Math can be used to introduce, model, reinforce, or extend instruction of the Texas Essential Knowledge and Skills (TEKS) for Mathematics. High-quality instruction must contain cognitive rigor and allow students to think and communicate about mathematics. *ThinkUp! Math* offers multiple opportunities for students to think critically, to apply mathematical reasoning and problem-solving skills, and to participate in standards-based learning activities.

One goal of Texas teachers is to plan and implement effective and engaging learning experiences that address the concepts and skills outlined in the TEKS. In addition to teaching math concepts and skills, teachers must address the seven Mathematical Process Standards (TEA, 2014b; 2017b). The mathematical process standards are located at the beginning of each grade level TEKS following *(b) Knowledge and Skills*. The process standards weave the content knowledge and skills together so that students may grow as problem solvers in order to use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and in every unit of *ThinkUp! Math,* providing students opportunities to acquire and demonstrate mathematical understanding.

ThinkUp! Math resources are provided to assist teachers with these instructional challenges. The student editions provide focused, aligned reinforcement and assessment of concepts as outlined in the TEKS for Mathematics (TEA, 2014a) as well as opportunities for critical thinking and communicating with mathematics. The teacher editions assist teachers in clarifying the standards, developing motivational and engaging lesson plans, employing strategies for high-quality instruction, implementing ongoing assessment, and meeting the needs of diverse learners.

Because mathematics is a universal subject, anyone who is a participating member of society must know basic mathematics. Students' mathematical achievement, however, is ultimately determined and limited by the opportunities they have had to learn. Mathematics is not restricted to a select group of students. "All students must learn to think

mathematically, and they must think mathematically to learn" (Kilpatrick, Swafford, and Findell, 2001). The RAND Mathematics Study Panel (2003) declared that it is essential that students develop math proficiency. The passing of Every Student Succeeds Act (ESSA), requires that academic assessments for "math and reading or language arts be administered annually in grades 3-8 and at least once in grades 9-12…" (Mandlawitz, 2016, p.1). The critical issue of accountability will continue with ESSA, but assessments will be used to help improve schools and inform instruction. The law allows the state and local levels the opportunity to create systems for accountability, resources, interventions, and teacher evaluation systems. The federal requirements of *Every Student Succeeds Act* mandate that all students participate in the state assessment program. For students to become proficient in mathematics and demonstrate their skills and mathematical knowledge on state assessments, they must participate in daily math instruction that offers a strong foundation with learning experiences aligned to the standards specified by the state. *ThinkUp! Math* can be used as an aligned tool for assessment of student progress. As students are introduced to and master the standards, teachers can monitor the progress of both individuals and whole groups.

The United States Department of Education (2004) shared that "the recent National Assessment of Educational Progress (NAEP, the Nation's Report Card) showed that 27% of eighth-graders could not correctly shade 1/3 of a rectangle and 45% could not solve a word problem that required dividing fractions." Philips (2007) offered statistics that indicated adults have difficulties with everyday applications of mathematics in the real world. Other research indicates that students and adults experience problems in foundational mathematical skills (Hecht, Vagi, & Torgeson, 2007). The 2015 Sixth International Mathematics and Science Study (TIMSS) released data that shows, on average, long-term improvement on the mathematics assessments for fourth and eighth grade students as compared to 49 other countries for grade four and 38 other countries for grade nine. As reported by a collaborative group with the document released by the US Department of Education and the National Center for Education Statistics (Provasnik, et al., 2016), results show the following: Fourth graders from the U.S. achieved the average score 539 which was higher than the average scores of other fourth graders in 34 education systems yet lower than the average scores in 10 education systems. Eighth graders from the United States had an average score of 518 which was higher than the scores of other eighth graders in 24 education systems and lower than the average scores of students in 8 education systems. This trend does bode favorably for the United States in that the trend toward growth is occurring in some areas. The report can be accessed to show tables and more specifics than shared in this document. Based on the findings, the evidence clearly shows mathematics literacy continues to be of concern in the United States. Therefore, it is understandable why the National Mathematics Advisory Council (NMAC) advocates attention be given to mathematics education in the United States. While some data indicate progress, there continues to be a need for the United States to focus on improvement in mathematics education. Thus, *ThinkUp! Math* is designed to support student academic achievement as measured by the State of Texas Assessment of Academic Readiness (STAAR®). Critical thinking and mathematical problem solving are integrated throughout this resource. *ThinkUp! Math* offers teachers and students a quality resource for increasing performance of both the student and the teacher in mathematics.

Acknowledging the needs of the user is crucial in designing educational products that are effective in improving teacher and student performance and resulting in academic achievement. Recognizing this, the Mathematics Product Development Team at Mentoring Minds acted to gather external input. In September of 2018 an opportunity was extended by Mentoring Minds to glean information from educators. A Mathematics Focus Group (2018), comprised of teachers and administrators currently using Mentoring Minds Motivation Math, assembled in east Texas for feedback on several topics. The major purpose was to preview and to provide feedback about the design and content of the new resource *ThinkUp! Math*. Sample math units from the teacher and student editions were available for previewing and gathering timely input. Discussion questions included the following: (1) What do you see as the greatest strength? (2) What do you see as a weakness? (3) Is there a component you would like to see added? (4) What feature or component stands out to you as engaging for students and teachers? (5) How do you think this resource will support teachers as they implement and students as they master the mathematics standards? (6) What support would you look for in a teacher edition for lesson planning? Comments given by the focus-group attendees were positive and specific about the new components, features, and layout. Repeated feedback indicated math educators were pleased to see elements included such as pre-assessments, teacher reflection, TEK clarification, learning targets, rigorous questioning stems, scaffolded questions at the beginning of each unit, and alignment to the standards. Other comments showed approval for suggested time frames in the teacher edition and for the addition of entry level problems and extended assessment in the student edition. A comment voiced by many educators showed support for the math misconceptions component which educators indicated is often a missing piece of units. Insight garnered from this meeting indicated that educators approved of the deeper focus on critical thinking and how it intertwined with the math content. All observations and comments yielded favorable responses about the new product design, coupled with feedback indicating how these elements contributed to improvements to a Mentoring Minds mathematics resource currently used or in use by the focus group attendees. The information collected was analyzed and used to improve *ThinkUp! Math*. In conclusion, *ThinkUp! Math* was described as “a true instructional tool” and a comprehensive, all-in-one product. Mentoring Minds advocates that gathering input of the customers *and* using the input to inform the development of resources must be a continual part of the process for producing educational tools of higher-quality.

ThinkUp! Math is a rigorous and relevant supplemental Levels 1-8 mathematics resource developed by Texas educators to integrate critical thinking and focused aligned reinforcement into classroom instruction. *ThinkUp! Math* addresses all readiness and supporting student expectations of the Texas Essential Knowledge and Skills (TEKS) for grades 3-8 and addresses all student expectations for grades 1-2. These student and teacher resources are designed to improve students’ problem-solving capabilities. With units addressing each assessed student expectation, students are empowered to make connections between mathematics and everyday life. *ThinkUp! Math* incorporates research-based strategies and pedagogically sound principles for teaching and learning. This mathematics product is designed to support and enhance the best practices for teaching the TEKS. *ThinkUp! Math* consists of active learning techniques (Braun et.al, 2017) rather than students practicing mathematics

rules silently or asking the teacher to tell them the solution. In addition, *ThinkUp! Math* is also based on the Gradual Release of Responsibility Model (Pearson and Galagher, 1983; Levy, 2007). While this model was associated early with reading, research indicates that this approach can be used successfully in other content areas including math and is associated with higher levels of student achievement. Students are guided through the learning process with multiple and varied opportunities for practice and application to achieve independent mastery of targeted student expectations.

Written to reflect the depth, rigor, and complexity of revised state assessments, *ThinkUp! Math* complements existing mathematics curricula and can serve as a reinforcement or intervention. *ThinkUp! Math* reflects deep alignment to the Texas Essential Knowledge and Skills (TEKS) and addresses the National Council of Teachers of Mathematics process standards. The teacher edition includes hands-on instructional activities appropriate for variety in groupings (i.e., whole group, small group, partners, individual) and suggested interventions. Every standard at every level, whether eligible for testing or not, is addressed. Essential academic vocabulary, and children's literature connections are also identified for the standards. In addition, manipulatives are woven into the instructional activities.

Student data from the Spring 2018 STAAR® Mathematics Summary Report (TEA, 2018b) demonstrate a range of scores for students in grades three through five. The total tested third grade students were 386,467. For the category Numerical Representations and Relationships, third graders answered 71% of items correctly or an average of 5.7 items out of 8; for Computations and Algebraic Relationships category, third graders answered 65% of items correctly or an average of 8.5 items out of 13; for Geometry and Measurement third graders answered 62% of items correctly or an average of 4.4 items out of 7; and for Data Analysis and Personal Financial Literacy, third graders answered 70% of items correctly or an average of 2.8 items out of 4.The total fourth grade students tested were 397,924. For the category Numerical Representations and Relationships, fourth graders answered 69% of items correctly or an average of 6.2 items out of 9; for Computations and Algebraic Relationships category, fourth graders answered 67% of items correctly or an average of 7.4 items out of 11; for Geometry and Measurement fourth graders answered 65% of items correctly or an average of 6.5 items out of 10; and for Data Analysis and Personal Financial Literacy, fourth graders answered 68% of items correctly or an average of 2.7 items out of 4. The total fifth grade students tested were 400,664. For the category Numerical Representations and Relationships, fifth graders answered 66% of items correctly or an average of 4.0 items out of 6; for Computations and Algebraic Relationships category, fifth graders answered 69% of items correctly or an average of 11.7 items out of 17; for Geometry and Measurement fifth graders answered 70% of items correctly or an average of 6.3 items out of 9; and for Data Analysis and Personal Financial Literacy, fifth graders answered 66% of items correctly or an average of 2.6 items out of 4.

Student data from the Spring 2018 STAAR® Mathematics Summary Report (TEA, 2018b) demonstrate a range of scores for students in grades six through eight. The total sixth grade students tested were 387,665. For the category Numerical Representations and Relationships, sixth graders answered 57% of items correctly or an average of 5.7 items out of 10; for Computations and Algebraic Relationships category, sixth

graders answered 55% of items correctly or an average of 8.3 items out of 15; for Geometry and Measurement sixth graders answered 54% of items correctly or an average of 3.2 items out of 6; and for Data Analysis and Personal Financial Literacy, sixth graders answered 52% of items correctly or an average of 3.7 items out of 7. The total seventh grade students tested were 350,802. For the category Probability and Numerical Representations, seventh graders answered 60% of items correctly or an average of 3.6 items out of 6; for Computations and Algebraic Relationships category, seventh graders answered 55% of items correctly or an average of 8.2 items out of 15; for Geometry and Measurement seventh graders answered 54% of items correctly or an average of 6.5 items out of 12; and for Data Analysis and Personal Financial Literacy, seventh graders answered 49% of items correctly or an average of 3.4 items out of 7. The total eighth grade students tested were 333,762. For the category Numerical Representations and Relationships, eighth graders answered 66% of items correctly or an average of 2.6 items out of 4; for Computations and Algebraic Relationships category, eighth graders answered 62% of items correctly or an average of 9.9 items out of 16; for Geometry and Measurement eighth graders answered 63% of items correctly or an average of 9.5 items out of 15; and for Data Analysis and Personal Financial Literacy, eighth graders answered 62% of items correctly or an average of 4.4 items out of 7.

While there have been increases in some reporting categories over the years, there are still some gaps. Several reasons might account for the lower range results. New TEKS for mathematics were first implemented in the 2014-2015 school year. When the TEKS were presented, they were written with rigor, and many math concepts were moved to different grade levels. The assessment in 2018 represents the fourth year that these TEKS for mathematics were assessed on STAAR. Gaps in learning may continue to exist due to instructional adjustments to higher levels of rigor with students being asked to think critically and apply mathematics processes rather than memorizing information. Also, results might be impacted because Spring 2018 was the second year for a shortened, more intense state assessment. While all TEKS should be a part of instruction each year, it appears that TEKS continued to need clarification for teachers to understand what the instruction should entail. Because the supporting TEKS rotate, this might cause a problem for teachers as they plan instruction. It is possible that teachers are not looking at ongoing student data to determine progress and non-progress so appropriate interventions or support is provided to students. As evidenced by these results, there appears to be a continued need for quality resources that advance deeper thinking while supporting the implementation of effective teaching and learning for mathematics content skills and knowledge and well as practice in applying mathematics processes. *ThinkUp! Math* provides a wealth of resources that address all the mathematics content standards for a given grade level while integrating problem solving, critical thinking, and communication skills. Opportunities for students are provided to connect mathematics to the everyday world and to other content areas. *ThinkUp! Math* offers instructional support with formative assessment, critical thinking, and reflection opportunities integrated throughout the instructional units, addressing all TEKS.

The Product Development Team for *ThinkUp! Math* developed this resource to help students in Levels 1-8 achieve mastery of identified Readiness and Supporting Standards. *ThinkUp! Math* is based on the Mathematics TEKS (TEA, 2014a), released

sample test items (TEA, 2015; 2016a; 2016c; 2017a; 2018a), TEA STAAR® Blueprints (TEA, 2016a), STAAR® Mathematics Resources (TEA, 2014c; 2017b), and information gleaned from conferences and individuals (Gutherie and Duncan, 2010; 2011). For Levels 1 and 2, students are provided learning experiences built around corresponding grade-specific standards. These experiences target the standards and provide repeated practice along with multiple opportunities to demonstrate learning, revealing progress toward mastery.

Each student edition unit in Levels 1-8 includes the following components: Getting Started (Learning Targets, Focus for 9 Traits of Critical Thinking™), Instruction (Concept Exploration, Vocabulary Mastery, Concept Development, Concept Application, Concept Practice), Extension (Math Challenge, Reflection on My Learning, Reflection on Critical Thinking), and Assessment, all which allow students to reflect and communicate their knowledge of mathematics. In Levels 2-8, all components are included with one exception: Online preassessments are offered. Within the unit the components are displayed throughout ten pages: one page is dedicated to the learning target and responding to prompts about the integration of two critical traits, six pages of reinforcement are provided for each math standard, an additional page for extending the concept or standard, and another two pages for the assessment. Grouws and Cebulla (2000) note that students need to be given both an opportunity to discover and invent new knowledge and an opportunity to practice what they have learned to improve student achievement. *ThinkUp! Math* presents multiple learning experiences in which to apply this finding; thus, the reason for the varied components.

Numerous studies indicate that increasing the amount of time spent in mathematics instruction is positively correlated with student achievement in mathematics. The 2001 National Research Council publication, "Adding It Up: Helping Children Learn Mathematics" states that significant time should be devoted to daily mathematics instruction in every grade of elementary and middle school. In addition, the 1999 Handbook of Research on Improving Student Achievement (Cawelti, 1999) states that a favorable relationship between total time allocated to mathematics and general student performance exists. A finding in The Nations' Report Card: Mathematics 2000, NAEP showed that the average scores of fourth and eighth graders generally increased as the amount of instructional time for mathematics increased. Grouws and Cebulla (2000) also concluded in their work that a positive relationship existed between total time allocated to mathematics and general mathematics achievement. Furthermore, the way in which time is utilized in mathematics class can be paramount to the degree of student achievement. *ThinkUp! Math* is arranged to allow teachers the flexibility in the allocation of time per component and/or per unit, pending the allocation of time requirements within the district or what is warranted by the needs of students, the intervention period cycle, or teacher discretion.

The following components are found in the teacher edition to ensure students reach high levels of mathematical performance based on the standards for mathematics: Getting Started (Clarifying the TEKS, Common Errors or Misunderstandings, Teacher-to-Teacher Tips, Focus for 9 Traits of Critical Thinking™), Instruction (Concept Exploration and Formative Assessment, Vocabulary Mastery and Formative Assessment, Literature Connection, Concept Development and Formative Assessment, Concept Application/Concept Practice, and Concept Check), Intervention (Activities and Formative Assessment), and Extension (Reflect on My Learning, Extending Student Thinking,

Home Connection, and Teacher Reflection). Support is found in related literature as to why these components are vital to the success of mathematics instruction and is noted in this document. Levels 3-8 teacher editions list the STAAR® Reporting Categories and Content and Process TEKS for each unit in the student edition. Suggested activities and interventions based on the Content TEKS for each of the units are located in 1-8 teacher editions. They are coded to the English Language Proficiency Standards or ELPS (TEA, 2007). Answer keys and vocabulary for each lesson are also included.

As indicated in the *Principles and Standards for School Mathematics* document (NCTM, 2000), assessment is essential in teaching and learning mathematics. In the publication compiled from several writers What We Know About Mathematics, Teaching, and Learning (McREL, 2010), it is reported that evaluation tools that have a close alignment with objectives are more beneficial for diagnosing and revising instructional needs. Due to accountability issues and in efforts to improve students' engagement and achievement, assessment is an essential component of *ThinkUp! Math.* The assessment focuses on the tested expectation(s) from whence the teacher can gather timely student information to readily and continuously maintain accountability for academic achievement standards before, during, and after the lesson. The *ThinkUp! Math Student Editions* offer assessment items in a variety of formats including selected-response, constructed-response, and open-ended griddable response. The *ThinkUp! Math Teacher Editions* offer a variety of formative assessments that can help teachers gauge student understanding and adapt instruction as needed. Data from assessments help teachers make informed decisions about student learning and about their own instructional practices.

Many of the pages within the unit can serve as formative assessment opportunities. Formative assessment or assessment for learning is crucial in schools today, as it involves students in the learning process and points to the next steps to be taken by students to advance their learning. When assessment is an integral part of mathematics instruction, it contributes significantly to students' mathematical learning (Stecker et al., 2005). Assessment should inform and guide teachers as they make instructional decisions. The tasks that teachers select for assessment convey messages to students about what kinds of mathematical knowledge and performance are valued. There are several pages in *ThinkUp! Math* which could be used as formative assessments that are interwoven throughout each unit. In the teacher edition formative assessments are embedded within the components. Selected-response items and constructed-response items are contained in Levels 1-8 student editions. All levels are interactive or require hands-on activities. Observation of students as they manipulate and interact with materials provides teachers with formative assessment data with which to adjust or adapt future instruction. Such activities also establish a foundation for the standard(s) being addressed in the unit. Concept Exploration and Vocabulary Mastery, two parts of the Instruction component, offer some open-ended format items. For the most part, Concept Check includes selected-response on the first page and open-response on the second page. The provision of open-ended items allows teachers to use students' responses to determine individual strengths and weaknesses and reasoning abilities. Teachers then have data to prescribe the depth of instruction and/or interventions required. Open-response questions are best practice classroom opportunities and have produced evidence showing positive effects in improving student performance. These types of questions provide

teachers with opportunities to better understand current knowledge, thinking, and comprehension of the concept displayed by students. When students show their work, teachers can diagnose problems such as error patterns in computation and reasoning. Research shows that this is not easily possible with a selected-response format. In Levels 6-8, assessment items reflect selected-response and griddable items. Griddable questions are a type of open-ended question used on all mathematic assessments that allow students opportunities to derive answers independently without the influence of answer choices. *ThinkUp! Math* features the same pattern for griddable items as included on STAAR®.

In the component Extension, Reflection on My Learning showcases journal prompts in all levels of student editions, offering authentic writing opportunities and, as lauded by research, serve as valuable instructional learning experiences for concept application to real-world settings. Open-ended problems are presented for students to solve using words, numbers, or pictures, and to follow up with written explanations. Mathematical concept prompts allow students to reflect and communicate their knowledge of mathematics. The mathematics journal prompt asks students to apply some aspect of the concept to a real-world setting or to examine the concept in relation to personal experiences. Students think about the concept as it applies to their own lives and communicate their thinking using the language of mathematics. The prompts in the Extension component and in the Getting Started components for *ThinkUp! Math* serve as formative assessment opportunities for students to express their thoughts and reasoning abilities as they transfer mathematical concepts across the disciplines, forming real-world connections.

Feedback from the variety of formative assessment tasks and from the reflection opportunities in *ThinkUp! Math* helps students know how to improve and what next steps to take. Other benefits will be students play prominent roles in setting goals, assume responsibility for their own learning, and become independent learners. Therefore, teachers can gather timely student information or data to readily and continuously maintain accountability for academic achievement standards in mathematics. Assessment for learning is a common occurrence within both student and teacher editions so that teaching can be adjusted, and learning can improve and grow. A Chart Your Success page is included in all levels 1-8 and is in the back of each student edition for each student to visually record and follow ongoing progress. The Assessment component pages used in conjunction with other measures can provide crucial information for the teacher in improving performance. Studies support the use of a variety of measures to gauge student achievement. Due to accountability, Mentoring Minds encourages teachers to maintain accurate and useful data as well as use a variety of assessment measures to form a more valid insight on where a campus, classroom, or student stands in mathematical performance. Effective and high-quality instruction is a result of using data to make informed decisions.

Assessment opportunities in *ThinkUp! Math* reflect flexibility and variety. A basic characteristic needed to become a proficient problem solver is flexibility. Flexibility develops through the expansion of knowledge required for solving nonroutine problems rather than just routine problems. Nonroutine problems (those not familiar to the problem solver) and transfer of problem solving require high level transfer, which is effortful and conscious (Salomon & Perkins, 1989), whereas routine problems involve less conscious attention and rely more on low level transfer. Routine problems are those in which the learner knows a correct solution method based on experience

and can reproduce it and apply it. Caution, given by experts, is that students can lose the ability to articulate and reflect on the reasoning they use in solving problems if they are exposed to mostly routine problems. Nonroutine problems require the learner to use productive thinking to create a way to understand and solve the problem since an immediate solution method is not known. A balance is needed between the time students spend practicing routine procedures and the time they devote to discovering new method solutions for nonroutine problems. There is no need for teachers to make a choice between which of these two type problems to use if students are to develop mathematical thinking power. Assessing the work of students in a problem-solving situation differs from a traditional method of determining the accuracy of computational skills. Open-ended problems can be solved using a variety of methods or the problems can have multiple responses. *ThinkUp! Math* incorporates a variety of assessment opportunities.

Critical thinking continues to be an important issue in education. In years past, attention focused on quality thinking as an important element of life success (Huitt, 1998; Thomas and Smoot, 1994). In the 1950s, Bloom found that 95% of the test questions developed to assess student learning required them only to think at the lowest level of learning, the recall of information. Similar findings indicated an overemphasis on lower-level questions and activities with little emphasis on the development of students' thinking skills (Risner, Skeel, and Nicholson, 1992). Hobgood, Thibault, and Walberg (2005) note that a large focus today is being given to the students and their abilities to think critically. Other educators agree on the importance of teaching students how to think critically and to reflect on their learning (Stobaugh, 2013a, 2013b; Love and Stobaugh, 2018). Based on what students are tasked to do, *ThinkUp! Math* clearly shows it is imperative for students to communicate their mathematical thinking coherently and clearly to peers, teachers, and others. Mentoring Minds development teams agree that the emphasis on thinking processes (Mentoring Minds, 2017) will lead to new levels of student performance. Thus, thinking is embedded into the learning experiences in the student editions and plays an important part in the teacher editions.

When solving mathematical equations, it is crucial for students to explain their thought processes. If the results are inaccurate, teachers can identify the precise point at which students deviated from using critical thinking. Thus, it is essential that classrooms promote critical thinking as part of the learning experiences in mathematics. The literature notes that when students use their critical thinking abilities integrated with content instruction, depth of knowledge can result. Teachers are encouraged to refrain from limiting instruction to lectures, rote memorization, and other strategies that exercise only lower levels of thought as opposed to incorporating those that build conceptual understanding (Bransford, Brown, and Cocking, 2000). Thus, according to a position statement issued by the National Council of Teachers of Mathematics (2013), to develop mathematical proficiency, students must be encouraged to communicate, explaining their thinking and reasoning abilities beginning in the early years. The models used to structure critical thinking throughout *ThinkUp! Math* are Revised Bloom's Taxonomy (Anderson et al, 2001), Webb's Depth of Knowledge (2002), and Hess' Cognitive Rigor Matrix for Mathematics (Hess, 2013). The framework used to develop the thinking traits is the 9 Traits of Critical Thinking™ (Mentoring Minds, 2017). These cognitive models were used by the product developers to stimulate and develop students'

higher order thinking skills and make extensions to the real world. More so, the 9 Traits of Critical Thinking™ provide direction for developing intellectual behaviors that are characteristic of strong critical thinkers.

Albert Einstein stated that education "is not the learning of the facts, but the training of the mind to think" (as cited in Frank, 1947, p. 185). Similarly, Margaret Mead (n.d.) commented, "Children must be taught *how to think,* not what to think." Some researchers wondered if critical thinking could be a predictor in any way. Butler, Pentoney, and Bong (2017) explored whether critical thinking can be a predictor of life events. These researchers collected data from persons who are shown to be strong thinkers. Findings from the study reveal fewer negative decisions are made by stronger thinkers in their personal lives. Based on their limited research and a wide range of growing literature that stated that critical thinking can be enhanced, these researchers advocate that critical thinking be addressed in instruction. The product development teams at Mentoring Minds believe that educators have an opportunity and a responsibility to equip students with the critical thinking skills and thinking traits that better organize thinking and transfer learning to new situations. Mentoring Minds purposefully creates resources for teachers and students that integrate thinking and grow skillful thinkers.

Two key areas in preparing students for college and career readiness are critical thinking and problem-solving skills (MetLife, 2011; Achieve, 2015). Based on an examination of top-performing global educational systems, a key identifier of successful systems is rigor (Ripley, 2013). Schools have been criticized for not adequately preparing students for the level of rigor they will encounter in college (Achieve, 2006). In 2011, only 25% of high school graduates taking the ACT successfully passed all four of the ACT's College Readiness Benchmarks, and 28% of high school students did not pass any of them. ACT predictions have been confirmed: nearly one third of students entering post-secondary education take remedial courses in one or more subjects because they lack the skills to take standard credit-bearing courses (National Center for Education Statistics, 2011). Moreover, research into the success rates of college students and high school seniors has shown that students' level of critical thinking is predictive of their grades or cumulative college grade point average (Facione, 1990a, 1990b; Sternberg, 2008).

In terms of employment, an overwhelming percentage of employers (93%) have indicated that job candidates' capacity to think critically, communicate clearly, and solve complex problems is more important than their college major (Association of American Colleges and Universities, 2013). When asked in 2015 how American public high schools could do a better job of preparing students for the expectations of college and the working world, college instructors and employers emphasized the need for critical thinking and problem-solving skills. This is especially true today, where new knowledge is rapidly accelerating, and information is instantly available. Students with critical thinking and problem-solving skills can interpret and evaluate what they read, see, and hear to effectively make the transition to college and career.

Educators, parents, and community members also agree that critical thinking and problem-solving skills are important skills for students. The findings of Project Tomorrow (2014), a survey of district administrators, teachers, parents, and community members, show critical thinking and problem-solving skills as essential skills needed by students for future success. As previously indicated, there is a connection between critical thinking skills and success in life—not just in college and the

workplace. Research has found that adults who scored higher on critical thinking assessments reported fewer negative life events. Possessing critical thinking skills helped the participants make positive life choices (de Bruin, Parker & Fischoff, 2007). This is echoed by Nisbett (2016), who states, "Schools cannot claim to prepare students for life unless they help students learn to reason effectively and to make choices that will improve their lives and the lives of others" (p. 28).

Encouraging and fostering thinking is central to student learning. In education, a shift from a focus on content to an emphasis on thinking skills is apparent. Thinking must be integrated with content to make meaning and deepen learning. Costa and Kallick (2009, p. 5) state that the standards "suggest that successful instruction in skillful thinking should be done *while* teaching subject matter instead of *in addition to* teaching subject matter. Thinking and subject matter content are neither separate from nor in opposition to each other. The implication is that a student cannot demonstrate mastery of any of these required standards without performing one or more important thinking skills." Thus, the resource *ThinkUp! Math* addresses all student expectations in the TEKS. Most units address one standard, although sometimes two or three standards are clustered together if applicable for teachers to instruct. During the instruction of a lesson, the focused content promotes deeper learning by encouraging students to share evidence of their thinking or reasoning for solutions—a vital part of developing math proficiency—rather than simply providing facts or a single answer.

State standards pave the direction for classroom instruction. Evidence of critical thinking is woven throughout the standards and the concepts required for each content area. At every grade level, all students must be taught to think critically and display behaviors that show that growth is being made in this arena. The problem that so many educators face is the "how." *How* do we teach students to become critical thinkers? *How* do we integrate that into the curriculum we teach? *How* do we assess that thinking is occurring? *How* do we guide students to share the responsibility in thinking critically? These questions and more appear to be a reality for school leaders and teachers. So, where do educators begin and what precisely do they teach when it comes to critical thinking? Some educators indicate they are ill prepared and do not know where to turn. Time is of the essence in today's classrooms, so quality resources that guide teachers are imperative. The solution offered by Mentoring Minds—*ThinkUp! Math*—equips teachers with tools they need. Teachers are provided support in understanding and interpreting TEKS with research-based thinking and instructional strategies through a section titled Clarifying the TEKS in the Getting Started component of the teacher edition. Teachers are also offered support in integrating critical thinking that is infused into each unit in the resource.

Expectations should be clear and communicated to students so that they know learning is more than the acquisition of information and skill and that discussions are not merely superficial. Only then will students understand what a thinking classroom looks like. Classroom tasks, routines, and assessments will make it transparent that in-depth thinking is required for success. Higher-order questions (e.g., Why is ___? What are you assuming when you say ___? What evidence can you offer to support ___? How might ___?) and thoughtful responses are desired. Students will readily observe that tasks, routines, and assessments are designed to elicit thinking and to transfer and make meaning. *ThinkUp! Math* translates thinking from content-area instruction into a wide variety

of situations that allow students to see relevance. Thinking expectations enable individuals to effectively evaluate the quality of thinking of others and self-assess their own thinking, determining individual progress and the improvement needed. When expectations for thinking are made explicit, evidence can be gathered, and judgments can be formed (Paul & Elder, 2000). With the emphasis on learning and thinking at the core of instruction within a school, the focus of the classroom shifts from acquisition of content to making meaning. Learning targets are visible to students in the Getting Started component of every unit followed by the two critical thinking traits that are a part of each unit in *ThinkUp! Math*.

Teachers must be specific in making expectations explicit for a thinking classroom if they want students to participate and succeed in a thinking environment. Classrooms where math is taught can be work cultures or thinking cultures. In work cultures, an emphasis is placed on students completing assignments, often at a low cognitive level. Thinking cultures nurture students' thinking skills (Ritchhart, 2002). Stobaugh (2013a) notes that teachers can train brains in a "thought-full" classroom just as people visit a gym to train their bodies to be stronger and more agile. Classrooms that reflect thinking climates encourage student questions and inquiries that focus on higher-order thinking and deepen learning experiences (Love and Stobaugh, 2018). *ThinkUp! Math* emphasizes a thinking environment and clarifies learning targets in each unit so expectations are explicit, ensuring that students know they are expected to be active learners.

Students should be taught the importance of thinking critically and how critical thinking skills impact their future success. It is recommended that students be taught that improvement in thinking skills is like improvement in any sport or hobby. Emphasize that the development of thinking takes commitment as well as practice, practice, practice. Students must also understand that learning how to think critically develops and improves over time. With ThinkUp! Math, teachers can ensure that students know the purpose or the reason behind every learning experience so the focus remains on the learning itself and not the work. Stobaugh (2013b, p. 137) states, "By establishing a focus on thinking, teachers can transform classrooms from mass-production classrooms with students able to answer fact-based questions to classrooms that embody real learning through thinking as students analyze, critique, and create."

Beyond acquisition of skills and creation of a culture that promotes thinking, there is another consideration that can impact deeper thinking—students should become aware of and learn to apply attributes or behaviors that strong thinkers exhibit. Research indicates there are specific behaviors that high-quality thinkers demonstrate. Effective thinkers and high-performing individuals do appear to portray certain characteristics (Goleman, 1995; Perkins, 1991). Costa and Kallick (2008, p. 16) report there are certain characteristics that successful individuals "such as lawyers, mechanics, teachers, entrepreneurs, salespeople, physicians, athletes, entertainers, leaders, parents, scientists, artists, and mathematicians" tend to exhibit when faced with solving problems. They define these identifiable characteristics as "habits of mind."

In 2017, a team of educators from Mentoring Minds generated a list of traits they have observed throughout their education careers that were indicative of students who exhibited skillful thinking and deeper levels of thought. Based on their varied backgrounds of teaching and leadership experiences, elementary and secondary levels of curricula expertise, a range

of 5–38 years working with children, observations of students, conversations with teachers, and 7 months of focused discussions, careful study, and deliberation, these educators collaboratively narrowed their lists to nine behaviors that students exhibited more times than not when thinking critically. Collectively, these nine behaviors (adapt, collaborate, communicate, create, examine, inquire, link, reflect, and strive) were entitled *9 Traits of Critical Thinking™ (Mentoring Minds, 2017)*. The nine traits, when explicitly taught, modeled, and practiced, can guide students in becoming more successful when engaging in cognitively demanding tasks and in social interactions at school and in life beyond the classroom. The traits should be emphasized in context with content-specific learning experiences that align to a trait during instruction. Focus for the 9 Traits of Critical Thinking is featured in the teacher edition and in the student edition. To understand the trait, the development outcomes and prompts that teachers might use to determine if students are exhibiting the traits are offered. Strategies for explicitly teaching the traits are also shared. Students are also provided information in the student edition stating the expectations for each trait focused within the unit and are provided reflection opportunities to assess how they use and apply the focus trait(s) in each unit. The traits help students become increasingly aware of thinking and more alert to mindful behaviors they can internalize.

A featured visual or icon depicts each of the *9 Traits of Critical Thinking™* in *ThinkUp! Math*. These traits are integrated into unit instruction using a combination of two focus traits. The unit's focus traits may differ since any combination or any order of traits can be accented. Although two are identified are in each unit in *ThinkUp! Math*, other traits could also be impacted if the teacher so desired. Traits were selected according to the type of thinking required by the math activities designed for students. By developing the nine traits in students and integrating the traits into the curriculum, teachers can impact academic success in thinking and learning. The identified critical thinking traits are basic to all learning at all levels and in all subject areas. Each trait contributes to the creation of a thoughtful environment that supports the development of skillful thinking. The information surrounding each trait helps teachers as they model, support, and monitor trait development. The open-ended questions surrounding the traits in the student edition require students to engage in reflective and evaluative thinking. The intent is to nurture trait development across the curriculum to guide students to internalize and to display all nine traits whenever thinking in academics, social interactions, or as situations warrant.

Marzano (2009) stressed the importance of a common language as it provides a framework or a way to talk about instruction at school. Just as educators use a shared language to discuss effective instruction in order to improve student learning, it stands to reason that to converse about critical thinking and its development, a language common to all should also exist. According to Walsh and Sattes, "A language of thinking promotes exactness and precision in expressing cognitive processing" (2011, p. 144). When a shared understanding is developed based on the common language of critical thinking, teachers can engage in deliberate conversations to make real-time adjustments in planning and engaging students in meaningful thinking experiences. By developing this knowledge base, teachers are given opportunities to improve their expertise in thinking and to better understand the kinds of practice opportunities needed to help students

grow as independent thinkers. Thus, *ThinkUp! Math* features a common thinking language about the nine critical thinking traits throughout each unit. Valuing a common language can ripple among the school community, causing all stakeholders (students, teachers, school leaders, parents, community leaders) to speak and understand the same thinking language.

When students are taught to practice better thinking in school and in their daily lives, they will become more successful in cognitive-demanding tasks and learn to value thinking throughout their lives. Practice and skillful application of each trait can result in students' actions becoming more productive and automatic when they encounter unknown or challenging situations in the classroom and in the real world. Learning how to think equips students with the ability to navigate challenging life circumstances. Throughout *ThinkUp! Math*, the traits are integrated into the instruction of the standard or concept. During math instruction, each unit promotes a productive critical thinking climate with students being aware of their growth to become better thinkers.

In the Extension component of each unit in the teacher edition, *ThinkUp! Math* includes Home Connection ideas to encourage teachers to invite and nurture parent engagement in mathematics education. Product developers recognize that teachers must support and encourage parent collaboration with students regarding mathematics. Teachers are provided activities per unit with which to cultivate parent involvement with their children by reinforcing previously introduced skills. Research concludes that productive collaboration and interaction with parents have a favorable impact on attitudes towards mathematics and student achievement (Barton et al., 2004). Parents can be significant contributors to the learning process. Opportunities for parents to be involved in their students' learning allow parents to show an interest in the students' work. Parent involvement helps parents become familiar with the content and the way students are learning (National Council of Teachers of Mathematics, 2000). When parents take time to provide home encouragement, students have another opportunity to apply and practice the mathematical concepts previously learned. Teachers and other educational leaders should consistently help students and parents to understand that an increased emphasis on the importance of effort is related to improved mathematics performance (The National Mathematics Advisory Panel, 2008).

Research indicates that the more parents are excited and involved in the learning of their children, the more successful a child can be academically. When schools cultivate partnerships and engage families in their children's education, author Constantino (2008) stated that student achievement can increase. In addition, Constantino noted that schools must continuously nurture relationships with parents by providing them with resources to help their children succeed in school. Constant attention in strengthening relationships lays the foundation for high-quality engagement. West (1985) and Weller (1999) indicate there are parent behaviors that can lead to effective schools. When parents show support, interest, and become involved the success rate of students can rise. Students in at-risk situations show an increase in grades, test scores, and academics when their parents become involved in instructional programs (Dolan, 1996). The activity ideas for parents in *ThinkUp! Math* provide varied opportunities within each unit to reach and engage parents.

Findings shared in the past appear to continue to be relevant today. Weller (1999) advocates that when schools and teachers treat parents with genuine concern and make them feel

important, welcome, and needed, parents are more apt to take active roles in supporting their children in academic achievement. Bagin and Gallagher (2001) note that communicating on a regular basis with parents can promote student learning and reduce attendance problems. The conclusions from an extensive research review on parent/family involvement programs are shared by Henderson and Mapp (2002) in the report *A New Wave of Evidence: The Impact of School, Family, and Community Connections on Student Achievement*. Henderson and Mapp concur with other researchers that a favorable and substantiated relationship exists between family involvement and student success, regardless of race/ethnicity, class, or parents' level of education. A key finding is that children of parents who are involved in home and in school settings show improved performance in school. Thus, when teachers implement the ideas from the Home Connection, parents have opportunities to support their children with meaningful and relevant applications to the previously taught concepts. The information given helps the parent and child build oral language through informal conversation. Simply written, parents are invited to support mathematical learning by asking questions, making relevant comments, or setting up other home learning activities to reinforce previously introduced concepts. Assignments, intended to be completed in class or at home, enhance students' understanding, skills, and proficiency in mathematics. Thus, teachers are encouraged to implement Home Connection ideas in the *ThinkUp! Math* Teacher Edition.

Research findings indicate that certain teaching strategies and methods are worth careful consideration as teachers strive to improve their mathematics instruction. Stigler and Hiebert (2004) advocate when the improvement of teaching methods becomes the focus, student performance will likely show increased positive results. Teacher and student interaction are key to improvement. Many students learn mathematical concepts best through the manipulation of concrete materials because it helps them to build a mental representation of the concept. Manipulatives provide concrete introductions to abstract ideas. Every student should have an opportunity to have adequate "hands on" experiences with appropriate manipulatives before engaging in pencil-and-paper activities. Textbooks and other printed resources show the pictorial and symbolic representations of mathematical concepts.

According to several studies, the use of manipulatives can enhance the cognitive process. Suydam and Higgins (1977) researched activity-based teaching approaches, including the use of manipulatives, in kindergarten through eighth grades. The conclusion reported was "...lessons using manipulative materials have a higher probability of producing greater mathematical achievement than do non-manipulative lessons." Findings revealed that manipulatives are effective no matter the achievement, ability, or socioeconomic levels of students. Manipulatives and pictorial representations produce higher achievement as opposed to only symbolic representations, as students can construct models to show their understanding of mathematical ideas or processes. This allows teachers to observe how students think or reason so that misconceptions can be corrected in a timely manner. It also offers students opportunities to demonstrate their learning other than with paper and pencil. The relationship between longevity and the use of manipulatives indicate positive findings of enjoyment, interest, and understanding, thus, increasing student engagement in mathematics (Sowell, 1989; Ruzic and O'Connell, 2001). When

students' interest grows, mathematical ability is affected and attitudes towards mathematics improve. Sutton and Krueger (2002) report that long-term usage of concrete materials seems to be positively related to increases in mathematical ability. Research by Grouws and Cebulla (2000) suggests that teachers use manipulative materials regularly in order to give students hands-on experiences in order to construct meaning for the mathematical ideas they are learning. A major benefit for students would be to use multiple types of manipulatives when learning mathematical concepts to ensure broader comprehension.

In *Curriculum and Evaluation Standards for School Mathematics*, the National Council of Teachers of Mathematics (NCTM, 1989) recommends the use of manipulatives in math education especially for elementary levels. In the revised document, *Principles and Standards for School Mathematics*, NCTM (2000) continues to place emphasis on the importance of manipulatives and supports the use of manipulatives in mathematics instruction. The book *Adding It Up: Helping Children Learn Mathematics* was released by the National Research Council in 2001. This book features a comprehensive review of research for mathematics education and includes this statement:

"The evidence indicates, in short, that manipulatives can provide valuable support for student learning when teachers interact over time with the students to help them build links between the object, the symbol, and the mathematical idea both represent" (p 354). Data gathered from classroom activities indicate that "when students are exposed to hands-on learning on a weekly rather than a monthly basis, they prove to be 72% of a grade level ahead in mathematics" (p 27). Based on an analysis of NAEP data, Wenglinsky (2000) concurs that interaction with manipulatives over time is crucial to help students make connections. Although studies report there is no one way to best teach mathematics, there is agreement on the value of manipulatives to promote understanding of mathematical concepts. Based on research, the National Council of Supervisors of Mathematics (2013) advocates the need to systematically integrate manipulatives into instruction to help students reach proficiency in mathematics. In *ThinkUp! Math* the use of concrete materials is not limited to only modeling by the teacher as each unit offers ways to encourage students to think and verbalize their thoughts while interacting with manipulatives.

Every student should have an opportunity to have adequate "hands on" experiences with appropriate manipulatives before engaging in pencil-and-paper activities. Textbooks and other printed resources show the pictorial and symbolic representations of mathematical concepts. It is highly recommended that all classrooms always have an assortment of manipulatives for student accessibility. If the same materials to teach multiple ideas can be used during each school year, then the amount of time to introduce the manipulatives can be shortened and students are helped to visualize and establish connections between ideas. This does not preclude a teacher from introducing other manipulatives but provides consistency with essential manipulatives utilized at more than one grade level. Thus, research and mathematical experts agree that the one essential component in a mathematics program should be the appropriate use of manipulatives. Moore (2013) shared numerous research findings supporting the use of manipulatives for teaching math. As noted earlier in the unit, the use of concrete materials should not be limited to teacher demonstrations. It is essential that students use materials in meaningful ways rather than in a fixed and restricted way that focuses on recall rather than on thought. Thus,

ThinkUp! Math supports the use of manipulatives and identifies manipulative-based activities throughout the teacher edition to accompany student addressed expectations.

Suggested literature that can be used for integrating lessons across the curriculum is noted in the teacher's edition. For Levels 1-6, literature selections are listed to help students make connections. Levels 7-8 literature selections are addressed within the activities. "Through the use of books, students see mathematics as a form of communication. It has been proven that children learn best when they can apply their learned knowledge from one subject to another" (National Council of Teachers of Mathematics, 1989). Problems that emerge from books make the mathematics relevant, engage the learners, and present meaningful contexts for establishing mathematical thinking. Integrating literature into mathematics is a technique that teachers might use to promote meaning of concepts, increase motivation, foster critical thinking, and improve problem solving skills (Haury, 2001; Mink & Fraser, 2005). *ThinkUp! Math* includes children's literature as excellent tools for making connections between the literature and mathematical concepts.

Literature can stimulate a variety of creative and critical thinking responses from the students, such as performing a skit from the story followed by mathematics-related problems. Problem-solving strategies, including acting it out, drawing a picture, and constructing a model using manipulatives, materialize quite readily from this type of activity. Evidence shows that books encourage thought-provoking discussions and reasoning in mathematics when questions are presented on higher thinking levels. Discussions are encouraged to build conceptual understanding. Thaiss (1986) advocates a mathematics and literature connection to strengthen student motivation and increase higher levels of engagement. *Becoming a Nation of Readers: The Report of the Commission on Reading* (Anderson, 1985) stresses the importance of the integration of reading. Mathematics lends itself easily as a communication tool and thus, works directly with reading to help students become successful learners.

Intervention activities that are found in the teacher edition specify focused learning experiences that teachers may incorporate during instruction to teach and reinforce the concepts found in the student edition. Activities are explicitly employed to develop skills and mathematical conceptual understandings accenting the focused TEK and other related TEKS. Many of these activities are hands-on and require some type of manipulative. Often, the activities are at the concrete or pictorial levels to allow students to form connections. When students lack comprehension and are found to need additional small group instructional interventions, then the experiences in the section Interventions are readily available. The Intervention activities are targeted to prerequisite concepts. They always present alternative methods of teaching a foundational skill necessary for mastery of a concept. Active instruction includes a wide range of instructional approaches: small groups, class discussion, concrete objects, hands-on experiences, reading, and writing. In *ThinkUp! Math*, teachers can ask students to think aloud, consider different options for solving problems, show evidence for the solution reached, and put their thoughts in writing. All these ways help students to organize their thinking and assist teachers in determining the level of understanding of mathematical concepts. Studies indicate that instruction which emphasizes active student engagement in hands-on opportunities improves attitudes toward math and indicates a positive effect on mathematics achievement. Evidence from

previously mentioned research demonstrates that a successful mathematics program must include time for students to practice what they are learning and experiences to perform the tasks for which they are to demonstrate competence. Intervention activities in *ThinkUp! Math* Teacher Editions support students in their quest for mastery of the standards, are useful tools in the general education classroom to explore traits to improve thinking and serve as resources for tiered interventions settings.

The intent of the Vocabulary Mastery component is to make learning meaningful, fun, and interesting. Students must understand vocabulary to understand the academic content they encounter in school. Stahl and Fairbanks (1986) revealed when specific vocabulary from academic subject areas is selected as the focus of instruction, the result was a 33 percent increase. Therefore, it appears when students are taught specific academic vocabulary in each subject area at each grade level, students have an excellent opportunity to acquire the academic background knowledge they need to understand the subject area content. Teaching content vocabulary using a systematic approach appears be a powerful tool for student success (Marzano & Pickering, 2005). Furthermore, research firmly documents that academic background knowledge influences academic achievement. Any intervention for the achievement of students should identify increasing students' content vocabulary knowledge through direct instruction as a leading priority (Marzano, 2004). While the vocabulary is merely identified for each unit at all levels, some of the suggested teaching activities and the suggested children's literature integrate vocabulary within instruction. Encountering vocabulary frequently to deepen the meaning is a finding upheld by Beck, McKeown and Kucan (2002) and Nagy (2005). Within each *ThinkUp! Math* unit, TEKS-specific vocabulary for the unit is featured with an accompanying activity and questions. Vocabulary activities are also offered in the teacher edition that develop or reinforce essential mathematics vocabulary. A separate section for glossary support at each grade level is included in the back of the teacher and student editions so that teachers can guide students to build the academic background knowledge they need to fully understand content or develop concept(s). Vocabulary associated with the trait as it applies to the standard or concept that is the focus of the lesson is also introduced and reinforced through open-ended questions at the beginning of the unit and through a written reflection toward the conclusion of the lesson.

Mentoring Minds seeks to understand the issues involved in teaching and learning mathematics. The National Research Council (2001) asserted that the performance of students in both reading and math at the conclusion of elementary school is an important predictor of their educational success. Northwestern (2007) examined data from six studies involving about 36,000 preschoolers. Results indicated that students who mastered early math and literacy skills will have success in math and reading as they move through the grades. However, researcher Greg Duncan reported that mastery of early math skills can predict future performance in math and reading, but the data appears to not support mastery of reading skills being a predictor of successful math performance. Students who have not mastered certain basic skills can expect to encounter problems in mathematics throughout their schooling and later. Summary statements such as these, other research findings, a review of mathematical literature, combined with recommendations from studies and observations from classroom experiences have yielded much knowledge about what works. With this wealth of information, *ThinkUp! Math* was

developed as a complement to an existing mathematics program for any grade or campus and as a guide for teachers in facilitating success in mathematics. The Mentoring Minds Product Development Team embraces the goal that all students receive a quality mathematics education that promotes deeper thinking.

The content of *ThinkUp! Math* focuses on the STAAR® Reporting Categories and the student expectations in the TEKS, ensuring that the product is appropriate, high-quality, and up-to-date. Frameworks for critical thinking and the 9 Traits of Critical Thinking help form questions and/or learning activities that stimulate and develop students' higher-order thinking skills. Examples of research-based techniques applied in the development of *ThinkUp! Math* include: standards-based instruction, active teaching, hands-on instruction, critical thinking, and formative assessment. The experiences, discussions, and review of the literature convinced the Mentoring Minds Product Development Team that resources for mathematics needed a change. Thus, the format for student and teacher editions were designed to help move mathematics forward so that teachers could incorporate the teaching of TEKS in mathematics on a higher level and develop within students the confidence they need to succeed. The literature on improving student performance in mathematics concludes that effective mathematics programs offer specific information on individual student performance for teachers, parents, and students; peer feedback and support; direct or explicit instruction; and real-world problems. *ThinkUp! Math* meets the criteria specified for improving student performance in mathematics.

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